Common Types of Polynomial Questions

Hello!

Congratulations on taking the first step towards mastery of **Polynomials**!

This resource was created to help you save time and effort when doing your A.Math revision, so you are clear on the different types of polynomial questions that commonly appear!

Our teachers have taken time and effort to curate questions, and we are certain this valuable resource will help you in your revision.

Please make sure you use it!

Should you still require some help after going through these questions, do feel free to drop us a WhatsApp! Our friendly centre manager will always be ready to help!

Here's wishing you all the very best for your O Level A.Math tests and exams.

Teachers of The Classroom

WhatsApp: 87493227

www.theclassroom.com.sg

Skills required:

A Solving equations

B Find constant given factor or divisor with remainder

C Finding remainder

D Finding polynomial

Cubic Identities	Remainder Theorem	Factor Theorem
$a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$	If f(*x*) is divided by $(ax-b)$, then the Remainder = $f\left(\frac{b}{a}\right)$	If $(ax-b)$ is a factor of f(*x*), then $f\left(\frac{b}{a}\right)=0$

Questions

1. The function f is defined by $f(x)=6x^3-kx^2+3x+10$, where k is a constant.

   i. Given that $2x+1$ is a factor of f(*x*), find the value of *k*. [2]

   ii. Using the value of *k* found in part **(i)**, solve the equation f(*x*) = 0. [4]

2. It is given that $f(x)=(x+h)^2(x-1)+k$, where *h* and *k* are constants and $h < k$. When f(*x*) is divided by $x+h$, the remainder is 6. It is given that f(*x*) is exactly divisible by $x+5$. State the value of *k* and show that $h = 4$. [4]

3. The polynomial $f(x)=px^3+3x^2+qx-6$ is divisible by x^2+x-6.

   i. Find the value of *p* and *q*. [4]

   ii. Find the remainder in terms of *x* when $f(x)$ is divided by x^2-1. [2]

4. The term containing the highest power of *x* and the term independent of *x* in the polynomial $f(x)$ are $2x^4$ and -3 respectively. It is given that $(2x^2+x-1)$ is a quadratic factor of $f(x)$ and the remainder when $f(x)$ is divided by $(x-1)$ is 4.

   i. Find an expression for $f(x)$ in descending powers of x, [5]

   ii. Explain why the equation $f(x)=0$ has only 2 real roots and state the values. [4]

5. Show that $2x^2+1$ is a factor of $2x^3-4x^2+x-2$. [2]

6. The expression $3x^3+ax^2+bx+4$, where a and b are constants, has a factor of $x-2$ and leaves a remainder of -9 when divided by $x+1$.

   i. Find the value of a and of b. [4]

   ii. Using the values of a and b found in part (i), solve the equation $3x^3+ax^2+bx+4=0$, expressing non-integer roots in the form $\frac{c\pm\sqrt{d}}{3}$, where c and d are integers. [4]

Suggested Solutions (Polynomials)

1	i. $f(-\frac{1}{2})=0$ $f(-\frac{1}{2})=6(-\frac{1}{2})^3-k(-\frac{1}{2})^2+3(-\frac{1}{2})+10$ $0=-\frac{6}{8}-\frac{1}{4}k-\frac{3}{2}+10$ $k=31$ ii. $3x^2-17x+10$ $2x+1\overline{)6x^3-31x^2+3x+10}$ $\underline{-(6x^3+3x^2)}$ $34x^2+3x$ $\underline{-(34x^2-17x)}$ $20x+10$ $\underline{-(20x+10)}$ 0 $f(x)=(2x+1)(3x^2-17x+10)$ $0=(2x+1)(3x^2-17x+10)$ $=(2x+1)(3x-2)(x-5)$ $x=-\frac{1}{2}, x=\frac{2}{3}, x=5$		
2	$f(-h)=6$ $(-h+h)^2(-h-1)+k=6$ $k=6$ $f(-5)=0$ $0=(-5+h)^2(-5-1)+6$ $-6=(-5+h)^2(-6)$ $1=(-5-h)^2$ $1=25+10h+h^2$ $h=6\,(\text{rej}, h<k)\ or\ h=4$		

3	Solution (i) $x^2+x-6=(x-2)(x+3)$ By the factor thm, $f(2)=0$ $p(2)^3+3(2)^2+q(2)-6=0$ factor thm $8p+2q+6=0$ $4p+q=-3$ ....(1) $f(-3)=0$ $p(-3)^3+3(-3)^2+q(-3)-6=0$ factor thm $-27p-3q+21=0$ $9p+q=7$ ....(2) Solve (1) and (2); $p=2,\ q=-11$ (ii) Using $x^2=1$, $f(x)=2x^3+3x^2-11x-6$ $=2x^2(x)+3x^2-11x-6$ $=2x+3-11x-6$ $=-9x-3$		
4	**(i)** $f(x)=(2x^2+x-1)(x^2+bx+3)$ $f(1)=4$ $2(4+b)=4$ $b=-2$ $f(x)=(2x^2+x-1)(x^2-2x+3)$ $=2x^4-4x^3+6x^2+x^3-2x^2+3x-x^2+2x-3$ $=2x^4-3x^3+3x^2+5x-3$ **(ii)** $f(x)=(2x^2+x-1)(x^2-2x+3)$ $=(2x-1)(x+1)(x^2-2x+3)$ $(2x-1)(x+1)(x^2-2x+3)=0$ $x=\frac{1}{2}$ or $x=-1$ $x^2-2x+3=0$ $D=(-2)^2-4(1)(3)=-8<0$ $\therefore f(x)=0$ has only 2 real roots (Shown)		
5	$2x^3-4x^2+x-2=(2x^2+1)(x-2)$ Since there is no remainder, it is exactly divisible.		

6	i. $f(x) = 3x^3 + ax^2 + bx + 4$ x-2 is a factor $f(2) = 0$ $3(8) + 4a + 2b + 4 = 0$ $4a + 2b + 28 = 0$ $2a + b + 14 = 0$---------(1) $f(-1) = -9$ $-3 + a - b + 4 = -9$ $a - b = -10$---------(2) (1)+(2) $3a = -24$ $a = -8$ Sub into (2) $-8 - b = -10$ $b = 2$ ii. $x - 2 \overline{)\, 3x^3 - 8x^2 + 2x + 4}$ quotient $3x^2 - 2x - 2$ $3x^3 - 6x^2$ $-2x^2 + 2x$ $-2x^2 + 4x$ $-2x + 4$ $-2x + 4$ $3x^3 - 8x^2 + 2x + 4 = 0$ $(x - 2)(3x^2 - 2x - 2) = 0$ $x - 2 = 0$ $3x^2 - 2x - 2 = 0$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times -2}}{2 \times 3}$ $x = \frac{2 \pm \sqrt{28}}{6}$ $x = \frac{2(1 \pm \sqrt{7})}{6}$ $x = 2$ $x = \frac{1 \pm \sqrt{7}}{3}$		