

## Common Types of Partial Fraction Questions

Hello!

Congratulations on taking the first step towards mastery of Partial Fractions!

This resource was created to help you save time and effort when doing your A.Math revision, so you are clear on the different types of Partial Fraction questions that commonly appear!

Our teachers have taken time and effort to curate questions, and we are certain this valuable resource will help you in your revision.

Please make sure you use it!

Should you still require some help after going through these questions, do feel free to drop us a WhatsApp! Our friendly centre manager will always be ready to help!

Here's wishing you all the very best for your O Level A.Math tests and exams.

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**Types of questions:**

- A Proper fractions
- B Improper fractions

| <b>Partial Fractions</b>   |   |  |
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| <p style="color: red; margin: 0;"><b>Start all partial fraction questions by checking if the fraction is proper!</b></p> <p>(Power in numerator must be smaller than power in denominator!)</p> <ul style="list-style-type: none"> <li>If proper...                             <p style="margin-left: 20px;"><b>Step 1: Check if the denominator can be factorised</b>, if it can, <b>FACTORISE IT FIRST!!! (Important Tip!)</b></p> <p style="margin-left: 20px;">e.g. if the base is... <math>(x^2 - 9)(x + 3)</math>,</p> <p style="margin-left: 20px;">factorise it so it becomes <math>(x + 3)(x - 3)(x + 3)</math>,</p> <p style="margin-left: 20px;">which can then be written as <math>(x - 3)(x + 3)^2</math></p> <p style="margin-left: 20px;"><b>Some students will mistake this as a quadratic when in fact, it is repeated!</b></p> <p style="margin-left: 20px;"><b>Step 2: Next, do partial fractions!</b></p> </li> <li>If improper → long division first! Then use <math>\frac{\text{remainder}}{\text{divisor}}</math> to carry out partial fractions!</li> </ul> |   |  |
| <p><u>Type 1: Linear</u></p> <p>e.g. <math>\frac{5x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}</math></p>  | <p><u>Type 2: Repeated</u></p> <p>e.g. <math>\frac{x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}</math></p> | <p><u>Type 3: Quadratic</u></p> <p>e.g. <math>\frac{x^3 - 3x - 13}{(x^2 + 4)(x - 1)} = \frac{Ax + b}{x^2 + 4} + \frac{C}{x - 1}</math></p> |

## Questions

1. Express  $\frac{3x^3 - 5}{x^2 - 1}$  in partial fractions. [5]
2. Given that  $\frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x} = ax + b + \frac{c}{2x^2 - x}$ , where a, b and c are integers, express  $\frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x}$  in partial fractions. [5]
3. Express  $\frac{x+1}{x(x+3)^2 - (x+3)^2}$  in partial fractions. [5]
4. i. Factorise completely the cubic polynomial  $2x^3 - 11x^2 + 12x + 9$ . [3]
- ii. Hence, express  $\frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9}$  in partial fractions. [5]

Suggested solutions (Partial Fractions)

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|----------|--|
| <p>1</p> | <p>By long division [M1]</p> $\frac{3x^3 - 5}{x^2 - 1} = 3x + \frac{3x - 5}{x^2 - 1}$ $\frac{3x - 5}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ $3x - 5 = A(x-1) + B(x+1)$ <p><math>x = 1: 3(1) - 5 = 2B</math><br/> <math>B = -1</math></p> <p><math>x = -1: -3 - 5 = -2A</math><br/> <math>A = 4</math></p> $\therefore \frac{3x^3 - 5}{x^2 - 1} = 3x + \frac{4}{x+1} - \frac{1}{x-1}$   |
| <p>2</p> | <p>Using long division, <math>\frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x} = 3x - 6 - \frac{5}{2x^2 - x}</math></p> <p>Let <math>\frac{-5}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}</math></p> $-5 = A(2x-1) + Bx$ <p>Put <math>x = 0: A = 5</math></p> <p>Put <math>x = \frac{1}{2}: \frac{1}{2}B = -5</math><br/> <math>B = -10</math></p> $\therefore \frac{6x^3 - 15x^2 + 6x - 5}{2x^2 - x} = 3x - 6 + \frac{5}{x} - \frac{10}{2x-1}$ |
| <p>3</p> | $\frac{x+1}{x(x+3)^2 - (x+3)^2} = \frac{x+1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$ $x+1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$ <p>.</p> <p>.</p> $\frac{x+1}{x(x+3)^2 - (x+3)^2} = \frac{1}{8(x-1)} + \frac{1}{8(x+3)} + \frac{1}{2(x+3)^2}$  |

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| <p><b>4i</b></p> | <p>Let <math>f(x) = 2x^3 - 11x^2 + 12x + 9</math><br/> <math>f(3) = 0</math><br/> <math>\therefore (x - 3)</math> is a factor of <math>f(x)</math>.</p> <p><math>f(x) = (x - 3)(2x^2 - 5x - 3)</math><br/> <math>= (x - 3)^2(2x + 1)</math></p>   |
| <p><b>ii</b></p> | <hr/> <p><math display="block">\frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9} = 3 + \frac{-x + 24}{(x - 3)^2(2x + 1)}</math></p> <p><math display="block">\frac{-x + 24}{(x - 3)^2(2x + 1)} = \frac{A}{2x + 1} + \frac{B}{x - 3} + \frac{C}{(x - 3)^2}</math></p> <p><math>-x + 24 = A(x - 3)^2 + B(x - 3)(2x + 1) + C(2x + 1)</math></p> <p>➔ Using substitution/comparing coefficient<br/> <math>A = 2, B = -1, C = 3</math></p> <p><math display="block">\therefore \frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9} = 3 + \frac{2}{2x + 1} - \frac{1}{x - 3} + \frac{3}{(x - 3)^2}</math></p> |