Common Types of Binomial Questions

Hello!

Congratulations taking the first step toward mastery of Binomial Theorem!

This resource was created to help you save time and effort when doing your A.Math revision, so you are clear on the different types of A.math questions that commonly appear!

Our teachers have taken time and effort to curate questions, and we are certain this valuable resource will help you in your revision.

Please make sure you use it!

Should you still require some help after going through these questions, do feel free to drop us a WhatsApp! Our friendly centre manager will always be ready to help!

Here's wishing you all the very best for your O Level A.Math tests and exams.

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Binomial Theorem (3 Most Common Types of Exam Questions)

Type 1 - Expansion i. Find the first 3 terms… ii. Find the coefficient of x^n where n can be any whole number… To solve, i. just apply **expansion** formula! ii. must be familiar with "Laws of indices" and find out which term you need to multiply!	Type 2 - Finding Specific Terms i. Find the term independent of ***x***, ii. Find the coefficient of x^n where n can be any whole number… (this is different from type 1. Type 1 will usually have some expansion in part (a) that is related!) To solve, i. apply **general term** formula! Similarly, good to be familiar with "Laws of Indices".	Type 3 - Unknown power n. To solve this type of question successfully, you need to know how to expand $\binom{n}{r}$. e.g. $\binom{7}{2}=\frac{7\times 6}{1\times 2}$; $\binom{9}{3}=\frac{9\times 8\times 7}{1\times 2\times 3}$ so… $\binom{n}{2}=\frac{n(n-1)}{1\times 2}$
e.g. a. Find the first 3 terms of the expansion $(2-\frac{x}{3})^7$. b. Find the coefficient of x^3 in $(3x^2+4x-2)(2-\frac{x}{3})^7$. (Need to think which terms when multiplied will give power 3!)	e.g. 1. In the expansion of $\left(x^3-\frac{2}{x^2}\right)^{10}$, find the term in x^{10}. 2. The coefficient of $\frac{1}{x^2}$ in the expansion of $(ax-\frac{2}{x^2})^{10}$ is 336. Find the value of a.	e.g. 1. In the expansion of $(2+3x)^n$, the coefficients of x^3 and x^4 are in the ratio 8 : 15. Find the value of n. 2. Given that $(1+ax)^n=1-12x+63x^2+\ldots$, Find the value of *a* and *n*.

Questions

1i. Write down and simplify the first four terms in the expansion $(2x - \frac{p}{x^2})^5$ in descending powers of x, where p is a non-zero constant. [3]

ii. Given that the coefficient of x^{-1} in the expansion $(4x^3 - 1)(2x - \frac{p}{x^2})^5$ is $-160p^2$, find the value of p. [4]

2i. Find the coefficient of x^4 in the expansion of $(6 - x^2)^5(2x^2 + \frac{1}{3})$. [4]

ii. In the expansion of $(2 + x)^n$, the ratio of the coefficients of x and x^2 is $2:3$. Find the value of n. [5]

3. Find the term independent of x in the expansion of $2x(2x - \frac{1}{x^2})^8$. [4]

b. The first 3 terms in the binomial expansion $(1 + kx)^n$ are $1 + 5x + \frac{45}{4}x^2 + \ldots$. Find the value of n and of k. [4]

Suggested solutions

1.

$$(2x)^5+5(2x)^4\left(-\frac{p}{x^2}\right)+\binom{5}{2}(2x)^3\left(-\frac{p}{x^2}\right)^2+\binom{5}{3}(2x)^2\left(-\frac{p}{x^2}\right)^3+\ldots$$

$$=32x^5-80px^2+\frac{80p^2}{x}-\frac{40p^3}{x^4}+\ldots$$

ii.

$$(4x^3-1)(2x-\frac{p}{x^2})^5$$

For terms with x^{-1},

$$\ldots+4x^3\left(-\frac{40p^3}{x^4}\right)+(-1)\frac{80p^2}{x}+\ldots$$

$$=-\frac{160p^3}{x}-\frac{80p^2}{x}+..$$

Since coefficient is $-160p^2$,

$$-160p^2=-160p^3-80p^2$$

$$-80p^2+160p^3=0$$

$$p^2(-80+160p)=0$$

$$p=0, p=0.5$$

2.

$$\left(6-x^2\right)^5=\binom{5}{0}(6)^5\left(x^2\right)^0-\binom{5}{1}(6)^4\left(x^2\right)^1+\binom{5}{2}(6)^3\left(x^2\right)^2+\ldots.$$

$$=7776-6480x^2+2160x^4+\ldots.$$

$$\text{Coefficient of } x^4=(-6480)(2)+(2160)\left(\frac{1}{3}\right)$$

$$=-12960+720$$

$$=-12240$$

ii.

For x term, $r = 1$

$$T_2 = \binom{n}{1}\left(2^{n-1}\right)x$$
$$= \frac{2^n(n)}{2}x$$

For x^2 term, $r = 2$

$$T_3 = \binom{n}{2}\left(2^{n-2}\right)x^2$$
$$= \frac{2^n(n)(n-1)}{8}x^2$$

$$\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{\frac{2^n(n)}{2}}{\frac{2^n(n)(n-1)}{8}} = \frac{2}{3}$$
$$\frac{2^n(3n)}{2} = \frac{2^n(n)(n-1)}{4}$$
$$2^n(6n) = 2^n(n)(n-1)$$
$$2^n(n)(n-1) - 2^n(6n) = 0$$
$$2^n(n)\left[(n-1)-6\right] = 0$$

$2^n = 0$ (reject as $2^n > 0$)

$n = 0$ (reject as $n \neq 0$)

$n = 7$

3.

(a) For $\left(2x-\frac{1}{x^2}\right)^8$, $T_{r+1}=\binom{8}{r}(2x)^{8-r}\left(-\frac{1}{x^2}\right)^r$

For x^{-1}, $8-r-2r=-1$

$r=3$

Coefficient of $x^{-1}=\binom{8}{3}(2)^5(-1)^3$ = -1792

Term independent of x in $2x\left(2x-\frac{1}{x^2}\right)^8=-3584$.

(b) $(1+kx)^n=1+\binom{n}{1}kx+\binom{n}{2}k^2x^2+....$

$=1+nkx+\frac{n(n-1)k^2}{2}x^2+...$

Comparing coefficients : $nk = 5$ ……….. (1)

$\frac{n(n-1)k^2}{2}=\frac{45}{4}$

$2n^2k^2 - 2nk^2 = 45$……….. (2)

Subst (1) in (2) : $50 - 10k = 45$

$\therefore k=\frac{1}{2}$ and $n = 10$